ON FINITE GROUPS ISOSPECTRAL TO SIMPLE LINEAR GROUPS

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Given a finite group G, denote by $\omega(G)$ the *spectrum* of G, i. e., the set of its element orders. We call finite groups G and H *isospectral* if $\omega(G) = \omega(H)$. Let $h(G)$ be the number of pairwise nonisomorphic groups isospectral to G. Group G is called *recognizable* (by spectrum) if $h(G) = 1$, *almost recognizable* if $h(G) < \infty$, and *non-recognizable* if $h(G) = \infty$. Since every finite group with a nontrivial normal soluble subgroup is non-recognizable (see [1, Corollary 4] and [2, Lemma 1]), of prime interest is the recognition problem for nonabelian simple groups. It turned out that many of nonabelian finite simple groups are recognizable or at least almost recognizable. Recently it was proved in [3] the following

Theorem 1. *Suppose that $L \simeq PSL_n(q)$ or $L \simeq PSU_n(q)$ and $n \geq 45$. Then a finite group isospectral to L is isomorphic to a group G with $L \leq G \leq AutL$. In particular, there are only finitely many pairwise non-isomorphic finite groups G with $\omega(G) = \omega(L)$.*

It follows from this theorem that simple linear and unitary groups of sufficiently large dimension are almost-recognizable. We continue investigation of the recognition problem for simple linear and unitary groups and prove

Theorem 2. *Suppose that $L \simeq PSL_n(q)$ or $L \simeq PSU_n(q)$ and $27 \leq n \leq 44$. Then a finite group isospectral to L is isomorphic to a group G with $L \leq G \leq AutL$. In particular, there are only finitely many pairwise non-isomorphic finite groups G with $\omega(G) = \omega(L)$.*

The work is supported by Russian Science Foundation (project 14-21-00065).

References

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